SETS IN MAPLE

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The book *A Gentle Introduction to Group Theory*, by Bana Al Subaiei & Muneerah Al Nuwairan in its final chapter has a description of a software package called Sage which will do calculations in set and group theory. However, Sage appears to run only under Linux, and since I'm a devoted Windows user, I can't run it. I thought it would be useful to see how my mathematical program of choice, Maple, can do similar calculations.

In this post, we'll look at the basic operations that can be performed on sets. A set can be defined by listing its elements within curly braces. Set elements can be numbers as in (left-justified lines are the code to be typed in to Maple, and centred lines are Maple's response):

```
W := {1, 2, 3, 4, 5};
                    W := {1, 2, 3, 4, 5}
X := {3, 5, 6, 7};
                    X := {3, 5, 6, 7}
```

To define the empty set, use an empty pair of braces:

```
V := {};
```

Maple uses the empty set symbol $\emptyset$ to represent an empty set, but this symbol can't be entered in Latex's program code style.

To define a sequence of integers, use the notation:

```
U := {`$`(1 .. 10)};
          U := {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
```

Note that the $ sign here is surrounded by the backquote character ‘ and not the regular apostrophe.

It's also possible to define sets with any text as elements as in

```
Pets1 := {canary, cat, dog, gerbil}
Pets2 := {cat, gerbil, hamster}
```

The basic set operations are done as follows. Each operation is implemented using its textual form. So we have

```
W intersect X;
                         {3, 5}
W union X;
                {1, 2, 3, 4, 5, 6, 7}
W minus X;
                      {1, 2, 4}
X minus W;
                         {6, 7}
```

There is no special notation in Maple for a universal set, since the composition of a universal set depends on the application. Here, we'll take the set U above to be the universal set. The complement of a set is then found using the **minus** operator, so we have

```
U minus W;
                    {6, 7, 8, 9, 10}
U minus X;
                 {1, 2, 4, 8, 9, 10}
U minus V;
          {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
```

The identities described in an earlier post can be verified in Maple.

```
W union W;
                    {1, 2, 3, 4, 5}
W intersect W;
                    {1, 2, 3, 4, 5}
U minus (U minus W);   # Complement of the complement
                    {1, 2, 3, 4, 5}
W union V;
                    {1, 2, 3, 4, 5}
W intersect V;
                          {}
W minus V;
                    {1, 2, 3, 4, 5}
W minus W;
                          {}
```

The symmetric difference $W \Delta X$ is the set of elements in either W or X, but not in $W \cap X$. Thus it is $W \cup X - (W \cap X)$. Fortunately, there is a Maple command for this: **symmdiff**. To check it gives the correct answer, we compare the two ways of calculating $W \Delta X$:

```
(W union X) minus (W intersect X);
```

```
                              {1, 2, 4, 6, 7}
symmdiff(W, X);
                              {1, 2, 4, 6, 7}
```

The distributive laws can be tested:

```
Y := {2, 4, 6, 8, 10};
                         Y := {2, 4, 6, 8, 10}
W union (X union Y);
                    {1, 2, 3, 4, 5, 6, 7, 8, 10}
(W union X) union Y;
                    {1, 2, 3, 4, 5, 6, 7, 8, 10}
W intersect (X intersect Y);
                                     {}
(W intersect X) intersect Y;
                                     {}
```

The distributive law for the symmetric difference is a bit more complicated. We should have $W\Delta(X\Delta Y) = (W\Delta X)\Delta Y$. Using **symmdiff** we can verify this:

```
symmdiff(W, symmdiff(X, Y));
                              {1, 7, 8, 10}
symmdiff(symmdiff(W, X), Y);
                              {1, 7, 8, 10}
```

We can test De Morgan's laws, where the superscript c indicates the complement:

$$(X \cup Y)^c = X^c \cap Y^c \tag{1}$$

$$(X \cap Y)^c = X^c \cup Y^c \tag{2}$$

Using U as the universal set, we get:

```
U minus (X union Y);
                                  {1, 9}
(U minus X) intersect (U minus Y);
                                  {1, 9}
U minus (X intersect Y);
                    {1, 2, 3, 4, 5, 7, 8, 9, 10}
(U minus X) union (U minus Y);
                    {1, 2, 3, 4, 5, 7, 8, 9, 10}
```

Finally, we can visualize the relation between various sets using a Venn diagram. To do this, we need to include the *Statistics* package. For the sets W, X, Y and U above, we have

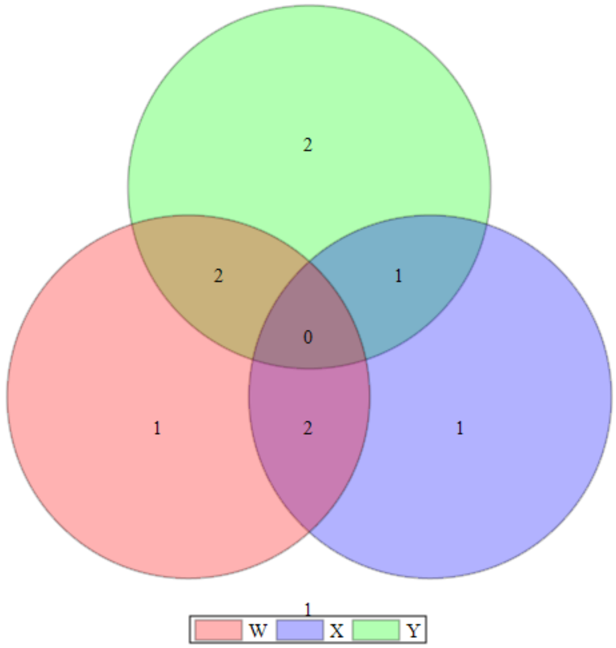

FIGURE 1. Venn diagram for sets W, X, Y with universal set U.

```
with(Statistics):
VennDiagram(W, X, Y, legend = ["W", "X", "Y"], universe = U);
```

This gives the Venn diagram in Fig. 1.

The numbers in each section of the diagram give the number of set elements in that section. Thus W is red and Y is green, and there are two elements in $W \cap Y$={2,4}.

PINGBACKS

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